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$\mathcal{N} = (4,4)$ type-IIA string theory on PP-wave background

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Abstract: We construct IIA GS superstring action on the ten-dimensional pp-wave background, which arises as the compactification of eleven-dimensional PP-wave geometry along the isometry direction. The background geometry has 24 Killing spinors and among them, 16 components correspond to the non-linearly realized kinematical supersymmetry in the string action. The remaining eight components are linearly realized and shown to be independent of x^+ coordinate, which is identified with the world-sheet time coordinate of the string action in the light-cone gauge. The resultant dynamical $\mathcal{N}$=(4,4) supersymmetry is investigated, which is shown to be consistent with the field contents of the action containing two free massive supermultiplets.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background, Extended Supersymmetry.

Contents

1. Introduction

In most cases, M theory is much easier to handle when it is compactified along the small circle, in which the theory becomes the weakly-coupled IIA superstring theory. In this case we can use full power of perturbative string theory to probe the model. It may be true even in the M theory on non-trivial background such as PP-wave geometry [1]–[5]. The PP-wave geometry for IIB string theory is maximally supersymmetric [6]. Furthermore, the IIB superstring theory on the PP-wave background reduces to free massive theory in the light-cone gauge [7] and thus many techniques in string theory on the flat Minkowski background can be adopted. The theory is used to probe the stringy nature of four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory in [8].

On the other hand, even though M theory on the eleven-dimensional PP-wave is also maximally supersymmetric, it is not easy to probe various aspects on the theory due to the lack of powerful tool as much as string theory. Matrix model on PP-wave background [8, 9] has interesting property such as the removal of flat directions due to the presence of mass terms. Subsequently, various aspects on matrix theory in PP-wave background has been investigated [11]–[17]. However it has time-dependent supersymmetry which does not commute with the Hamiltonian. The implication of the time-dependent supersymmetry is not clear. And still it may be desirable to study the model in the context of ten-dimensional string theory, more or less like the IIB string theory on PP-wave background.

In this paper we consider the type-IIA superstring theory on the following ten-dimensional PP-wave background:

$$\begin{aligned} ds^2 &= -2dx^+dx^- - A\left(x^I\right)\left(dx^+\right)^2 + \sum_{I=1}^{8}\left(dx^I\right)^2 \,, \\ F_{+123} &= \mu \,, \qquad F_{+4} = -\frac{\mu}{3} \,, \end{aligned} \tag{1.1}$$

where

$$A\left(x^{I}\right)=\sum_{i=1}^{4}\frac{\mu^{2}}{9}\left(x^{i}\right)^{2}+\sum_{i'=5}^{8}\frac{\mu^{2}}{36}\left(x^{i'}\right)^{2}. \tag{1.2}$$

As shown in the appendix, this geometry comes from the compactification of the eleven-dimensional PP-wave geometry along the isometry direction[18].[1] It has 24 supersymmetry, thus one may fear that the theory is not so simple compared to the case of maximally supersymmetric IIB string theory on PP-wave geometry. As we will show, 16 supersymmetry in target space is non-linearly realized on the string worldsheet theory. On the other hand, eight supersymmetry is linearly realized. This eight, so-called, dynamical supersymmetry, which is half to the maximally symmetric cases, turns out to be time independent. This means that each half of bosonic fields have the same mass as each half of fermionic fields, forming a supermultiplet. Thus the theory is described by the two-dimensional $\mathcal{N}$=(4,4) supersymmetric theory with two massive supermultiplets. The light-cone action of the IIA superstring on the above PP-wave geometry turns out to have only quadratic terms, the same as the case of the IIB superstring theory on the PP-wave, thus the theory seems to be almost as simple as the IIB case.

In section 2, we show that the PP-wave geometry admits 24 Killing spinors. In particular, we find the explicit form of the Killing spinors for later use. In section 3, we construct the ligh-cone superstring action on the above PP-wave background starting from the eleven dimensional supermembrane action on $AdS_4\times S^7$ in the PP-wave limit. The action becomes the one of two massive supermultiplets of (4,4) supersymmetry. In section 4, we identify the kinematical and dynamical supersymmetry on the worldsheet action. In particular, we give the transformation rules for the $\mathcal{N}=(4,4)$ worldsheet supersymmetry. In section 5, we draw some conclusions. In the appendix, we describe the compactification of the eleven-dimensional PP-wave which gives rise to the IIA PP-wave geometry and give some useful formula which is used in the main context.

Note added: While writing this manuscript, there appeared a paper [22] which treats the same subject. However, the action obtained in that paper seems to be non-supersymmetric while the background geometry is obviously supersymmetric.

2. Supersymmetry of IIA PP-wave geometry

The IIA PP-wave geometry eq. (1.1) has 24 Killing spinors. The original eleven-dimensional PP-wave background is maximally supersymmetric and therefore has 32 Killing spinors. Among them only 24 components survive after the compactification along x^9. This can be shown by counting the number of spinors which are invariant under the Lie derivative along x^9-direction.[2]

Here we directly compute the ten-dimensional Killing spinor equations and get the explicit expression for those Killing spinors. This will be needed in the study of super-

[1] This is different PP-wave geometry from the one used by Alishahiha et.al. in [19]. Their geometry comes from the T-duality of IIB PP-wave. See also refs. [20, 21] for similar construction.

[2] See ref. [18] for detailed derivation.

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symmetry in the IIA string theory and also it will illuminate the nature of supersymmetry more clearly. The Killing spinor equations come from the vanishing of supersymmetry variations of gravitino and dilatino fields, $\delta_\eta \psi_\mu = \delta_\eta \lambda = 0$. The general expressions for these supersymmetry transformation rules are given in the appendix.

The equation from the dilatino transformation rule, $\delta_\eta \lambda = 0$, reduces to the condition

$$\Gamma^+ \left(\Gamma^{12349} - 1\right) \eta = 0 \,. \tag{2.1}$$

From the gravitino transformation rule, we have another condition which is

$$\delta_\eta \psi_\mu = \left(\nabla_\mu + \Omega_\mu\right) \eta = 0 \,, \tag{2.2}$$

where the explicit expression for Ω_μ is given in the appendix. In our case at hand, $\Omega_\pm$ are

$$\begin{aligned} \Omega_- &= 0 \,, \\ \Omega_+ &= -\frac{\mu}{96}\Gamma^{123}\left(\Gamma^{-+}\left(9 - \Gamma^{12349}\right) + \left(15 - 7\Gamma^{12349}\right)\right) \,. \end{aligned} \tag{2.3}$$

For the spinors η which satisfy eq. (2.1), Ω_I's are given by

$$\begin{aligned} \Omega_i &= -\frac{\mu}{6}\Gamma^{123}\Gamma^+\Gamma^i \,, \\ \Omega_{i'} &= -\frac{\mu}{12}\Gamma^{123}\Gamma^+\Gamma^{i'} \,. \end{aligned} \tag{2.4}$$

In order to see the structure of the spinor more clearly and to solve the Killing spinor equations explicitly, we now introduce the following representations for SO(1, 9) gamma matrices:

$$\begin{aligned} \Gamma^0 &= -i\sigma^2 \otimes \mathbf{1}_{16} \,, \qquad \Gamma^{11} = \sigma^1 \otimes \mathbf{1}_{16} \,, \qquad \Gamma^I = \sigma^3 \otimes \gamma^I \,, \\ \Gamma^9 &= -\sigma^3 \otimes \gamma^9 \,, \qquad \Gamma^\pm = \frac{1}{\sqrt{2}}\left(\Gamma^0 \pm \Gamma^{11}\right) , \end{aligned} \tag{2.5}$$

where σ's are Pauli matrices, and $\mathbf{1}_{16}$ the 16×16 unit matrix. γ^I are the 16×16 symmetric real gamma matrices satisfying the Spin(8) Clifford algebra $\{\gamma^I, \gamma^J\} = 2\delta^{IJ}$, which are reducible to the $\mathbf{8_s} + \mathbf{8_c}$ representation of Spin(8). We note that, since we compactify along the x^9 direction, Γ^9 is the SO(1, 9) chirality operator and γ^9 becomes SO(8) chirality operator,

$$\gamma^9 = \gamma^1 \dots \gamma^8 \,.$$

Firstly, we consider the 16 component spinor which satisfies

$$\Gamma^+ \tilde{\eta} = 0 \,, \tag{2.6}$$

which is clearly a solution of the condition, eq. (2.1). With the above gamma matrix representation, eq. (2.5), the transformation parameter $\tilde{\eta}$ is of the form

$$\tilde{\eta} = \begin{pmatrix} 0 \\ \tilde{\epsilon} \end{pmatrix} \,. \tag{2.7}$$

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By plugging this into the condition eq. (2.2), we see from the $-$ and I components of the condition that $\tilde{\eta}$ is independent of the coordinates x^- and x^I, and hence $\tilde{\epsilon}$ may be at most a function of x^+.

The $+$ component of eq. (2.2), which is the only remaining nontrivial condition and specifies the light-cone time dependence of the spinor $\tilde{\epsilon}$, is read as

$$\partial_+ \tilde{\epsilon} = -\frac{\mu}{4}\gamma^{123}(1 - \frac{1}{3}\gamma^{12349})\tilde{\epsilon}\,. \tag{2.8}$$

Since $(\gamma^{12349})^2 = 1$, it is now convenient to decompose $\tilde{\epsilon}$ as $\tilde{\epsilon} = \tilde{\epsilon}^+ + \tilde{\epsilon}^-$ where $\tilde{\epsilon}^\pm$ are eigenstates of γ^{12349}:

$$\gamma_{12349}\tilde{\epsilon}^\pm = \pm\tilde{\epsilon}^\pm\,. \tag{2.9}$$

Then solving the eq. (2.8) for each of $\tilde{\epsilon}^\pm$ leads to

$$\begin{aligned} \tilde{\epsilon}^+ &= e^{-\frac{\mu}{6}\gamma^{123}x^+}\tilde{\epsilon}_0^+\,, \\ \tilde{\epsilon}^- &= e^{-\frac{\mu}{3}\gamma^{123}x^+}\tilde{\epsilon}_0^-\,, \end{aligned} \tag{2.10}$$

where $\tilde{\epsilon}_0^\pm$ are constant spinors with eight independent components. Therefore the 16 component Killing spinor $\tilde{\eta}$ considered so far depends only on x^+ and, as will be shown in the next section, correspond to 16 kinematical supersymmetry of the IIA string action.

Next we consider eight remaining Killing spinors, which will correspond to the dynamical supersymmetry of the string theory. First of all, from the condition, eq. (2.1), the transformation parameter of the dynamical supersymmetry can be written of the form,

$$\eta = \begin{pmatrix} \epsilon \\ \epsilon' \end{pmatrix}\,, \tag{2.11}$$

where ϵ should satisfy $\gamma^{12349}\epsilon = -\epsilon$. The x^- component of the Killing spinor equation (2.2) reduces to $\partial_-\eta = 0$, which means that η is independent of x^-. The x^I components of eq. (2.2) reduce to $(\partial_I + \Omega_I)\eta = 0$. Since $\Omega_I\Omega_J = 0$ for any $I, J = 1, \ldots, 8$ due to the fact that $(\Gamma^+)^2 = 0$, η depends on x^I, at most, linearly and thus is of the following form

$$\eta = \left(1 + x^I\Omega_I\right)\begin{pmatrix} \epsilon \\ 0 \end{pmatrix}\,, \tag{2.12}$$

where ϵ is independent of x^I as well as x^-. One can easily convince that this automatically satisfies $(\nabla_+ + \Omega_+)\eta = 0$ as well only if ϵ is independent of x^+. This means that the eight dynamical supersymmetry in the light-cone IIA superstring action, which corresponds to this Killing spinor, is independent of time and thus becomes genuine symmetry which commute with the Hamiltonian. The fermions in the matrix model in eleven dimensional PP-wave background have different masses from those of bosons as the supersymmetry in matrix model does not commute with the Hamiltonian. Now it is natural to expect to get the same mass content in bosons and fermions, which is indeed the case as will be shown in the next section.

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3. Light-cone IIA superstring action in plane wave background

The best way to get the IIA GS superstring action in the general background is the double dimensional reduction of the supermembrane action in eleven-dimensions[23]. In general, it is very complicate to get the full expression of the GS superstring action in the general background, which is up to 32th order in terms of the fermionic coordinate. However, in the case at hand, we know the full action of supermembrane in eleven-dimensional PP-wave background. This comes from the fact that eleven-dimensional PP-wave geometry can be thought as a special limit [4, 5] of $AdS_4 \times S^7$ geometry on which the full supermembrane action is constructed using coset method [24]. Therefore we start from the supermembrane action in eleven-dimensional PP-wave in the rotated coordinates eq. (A.4), given in the appendix. The supermembrane is wrapped on x^9, and becomes IIA superstring after the double dimensional reduction along x^9.

The general expression of the eleven dimensional supermembrane action is too complicated, especially for the double dimensional reduction. Thus we first simplify the action by fixing the fermionic κ-symmetry,

$$\Gamma^+\theta = 0\,. \tag{3.1}$$

Super elfbein of ref. [24] in this fixing condition has the following form:[3]

$$\begin{aligned} \hat{E}^a &= D\theta\,, \\ \hat{E}^{\hat{r}} &= \hat{e}^{\hat{r}} + \bar{\theta}\Gamma^{\hat{r}} D\theta\,. \end{aligned} \tag{3.2}$$

where $\bar{\theta} = i\theta^T\Gamma^0$ and $D\theta$ is the super-covariant one-form whose general expression is given by

$$D\theta = d\theta + \frac{1}{4}\hat{\omega}^{\hat{r}\hat{s}}\Gamma_{\hat{r}\hat{s}}\theta + \frac{1}{2!3!4!}\hat{e}^{\hat{r}}\left(\Gamma_{\hat{r}}{}^{\hat{s}\hat{t}\hat{u}\hat{v}} - 8\delta_{\hat{r}}^{[\hat{s}}\Gamma^{\hat{t}\hat{u}\hat{v}]}\right)\theta F_{\hat{s}\hat{t}\hat{u}\hat{v}}\,. \tag{3.3}$$

In the PP-wave background geometry, eq. (A.4), and in the κ-symmetry fixing condition, eq. (3.1), $D\theta$ reduces to

$$D\theta = d\theta - \frac{\mu}{4}\left(\Gamma^{123} + \frac{1}{3}\Gamma^{49}\right)\theta dx^+\,, \tag{3.4}$$

where Γ^{123} comes from the non-vanishing constant four-form field strength and Γ^{49} is due to the component of the spin connection $\hat{\omega}_+^{49} = -\mu/6$, which is related to the ten dimensional RR two-form field strength as $\hat{\omega}_+^{49} = F_{+4}/2$ under the Kaluza-Klein reduction. In the κ-symmetry fixing condition, the super three-form field is simplified as

$$\hat{B} = \frac{1}{2}\hat{e}^+ \wedge \hat{e}^i \wedge \hat{e}^j \hat{C}_{+ij} - \bar{\theta}\Gamma_{+\hat{I}} D\theta \wedge \hat{e}^+ \wedge \hat{e}^{\hat{I}}\,, \tag{3.5}$$

where $\hat{I} = 1, \ldots, 9$.

[3]The index notations adopted here are as follows: $M, N, \ldots$ are used for the target superspace indices while $A, B, \ldots$ for tangent superspace. As usual, a superspace index is the composition of two types of indices such as $M = (\mu, \alpha)$ and $A = (r, a)$. $\mu, \nu, \ldots$ $(r, s, \ldots)$ are the ten dimensional target (tangent) space-time indices. $\alpha, \beta, \ldots$ $(a, b, \ldots)$ are the ten dimensional (tangent) spinor indices. For the eleven dimensional case, we denote quantities and indices with hat to distinguish from those of ten dimensions. $m, n, \ldots$ are the worldsheet vector indices with values τ and σ. The convention for the worldsheet antisymmetric tensor is taken to be $\epsilon^{\tau\sigma} = 1$.

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Through the usual Kaluza-Klein reduction, the super zehnbein is related to the above super elfbein. Among the super zehnbein fields, E^r is the only what we need for the construction of the superstring action and is obtained from the relations $\hat{E}^r_\mu = \Phi^{-1/3} E^r_\mu$ and $\hat{E}^r_\alpha = \Phi^{-1/3} e^{\phi/6} E^r_\alpha$ where Φ is the super dilaton field given by $\hat{E}^9_9$ [25]. Their explicit expression in the component form is then

$$\begin{aligned} E^r_\mu &= e^r_\mu - \frac{\mu}{4} \delta^r_- \delta^+_\mu \bar{\theta} \Gamma^- \left(\Gamma^{123} + \frac{1}{3} \Gamma^{49} \right) \theta \,, \\ E^r_\alpha &= - \left(\bar{\theta} \Gamma^r \right)_\alpha \,. \end{aligned} \tag{3.6}$$

Having the component form of the ten dimensional super fields, it is ready to construct the type-IIA superstring action in the PP-wave background, eq. (1.1). In the superspace formalism, the IIA superstring action is written as follows [23]:

$$S_{IIA} = \frac{1}{2\pi\alpha'} \int d^2\sigma \left(-\frac{1}{2} \sqrt{-h} h^{mn} \Pi^r_m \Pi^s_n \eta_{rs} + \frac{1}{2} \epsilon^{mn} \Pi^A_m \Pi^B_n B_{BA} \right) \,, \tag{3.7}$$

where Π^r_m is the pullback of super zehnbein onto the worldsheet;

$$\Pi^r_m = \partial_m Z^M E^r_M \,.$$

By using the super zehnbein fields, eq. (3.6), each component of the pullback in the background, eq. (1.1), is expressed as

$$\begin{aligned} \Pi^+_m &= \partial_m X^+ \,, \\ \Pi^-_m &= \partial_m X^- + \partial_m X^+ \left[\frac{1}{2} A\left(X^I\right) - \frac{\mu}{4} \bar{\theta} \Gamma^- \left(\Gamma^{123} + \frac{1}{3} \Gamma^{49} \right) \theta \right] + \bar{\theta} \Gamma^- \partial_m \theta \,, \\ \Pi^I_m &= \partial_m X^I \,. \end{aligned} \tag{3.8}$$

On the other hand, the super three-form field, eq. (3.5), leads to the following Wess-Zumino term:

$$\begin{aligned} \frac{1}{2} \epsilon^{mn} \Pi^A_m \Pi^B_n B_{BA} &= \frac{1}{2} \epsilon^{mn} \partial_m Z^M \partial_n Z^N \hat{B}_{9NM} \\ &= -\epsilon^{mn} \partial_m X^+ \left(\bar{\theta} \Gamma^{-9} \partial_n \theta \right) . \end{aligned} \tag{3.9}$$

Plugging the expressions, Eqs. (3.8) and (3.9) into the super field action, eq. (3.7), we then finally get the κ-symmetry fixed action in the component form for the type-IIA superstring in the PP-wave background, eq. (1.1), which is obtained as

$$\begin{aligned} S_{IIA} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{mn} \Big[& -2\partial_m X^+ \partial_n X^- + \partial_m X^I \partial_n X^I - A\left(X^I\right) \partial_m X^+ \partial_n X^+ - \\ & - 2\partial_m X^+ \bar{\theta} \Gamma^- \partial_n \theta + \frac{\mu}{2} \partial_m X^+ \partial_n X^+ \bar{\theta} \Gamma^- \left(\Gamma^{123} + \frac{1}{3} \Gamma^{49} \right) \theta \Big] - \\ - \frac{1}{2\pi\alpha'} \int d^2\sigma \epsilon^{mn} \partial_m X^+ \bar{\theta} \Gamma^{-9} \partial_n \theta \,. & \end{aligned} \tag{3.10}$$

We now consider the bosonic light-cone gauge to get the action for the physical degrees of freedom. The equation of motion for X^+ is harmonic, the same as in the flat case, which means that the usual light-cone gauge, $X^+ \propto \tau$, is allowed. We then take the conventional light-cone gauge conditions as follows:

$$\begin{aligned} X^+ &= \alpha' p^+ \tau \,, \\ \sqrt{-h} &= 1 \,, \\ h_{\sigma\tau} &= 0 \,. \end{aligned} \tag{3.11}$$

where p^+ is the total momentum congugate to X^-. The last two conditions are for the fixing of the worldsheet diffeomorphisms and allow us to fix other worldsheet metric components consistently as

$$-h_{\tau\tau} = h_{\sigma\sigma} = 1 \,.$$

In this bosonic light-cone gauge choice, the superstring action, Eq. (3.10), is further simplified, and the light-cone string action, which we call S_{LC}, is read as

$$\begin{aligned} S_{LC} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \Bigg[& \eta^{mn} \partial_m X^I \partial_n X^I + \frac{m^2}{9} \left(X^i\right)^2 + \frac{m^2}{36} \left(X^{i'}\right)^2 + \\ & + \bar{\theta}\Gamma^- \partial_\tau \theta + \bar{\theta}\Gamma^{-9} \partial_\sigma \theta - \frac{m}{4} \bar{\theta}\Gamma^- \left(\Gamma^{123} + \frac{1}{3}\Gamma^{49}\right) \theta \Bigg] , \end{aligned} \tag{3.12}$$

where we have rescaled the fermionic coordinate as $\theta \to \theta/\sqrt{2\alpha' p^+}$. m is a mass parameter defined by

$$m \equiv \mu \alpha' p^+ \,,$$

which characterizes the masses of the worldsheet fields. We see that the light-cone gauge fixed action S_{LC} is quadratic in fields and thus describes a free theory as in the IIB case [7].

Though it is now obvious that S_{LC} describes the theory of free fields, the mass spectrum is still unclear due to the mass term of fermionic fields. Reading off the mass contents of the fermionic fields is necessary and important step, especially to see the supersymmetry of the action more clearly, which will be elucidated in the next section. We first rewrite the action in the 16 component spinor notation with $\theta^A = \frac{1}{2^{1/4}} \begin{pmatrix} 0 \\ \psi^A \end{pmatrix}$ (Superscript A denotes the SO$(1,9)$ chirality.), under which we have

$$\begin{aligned} S_{LC} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \Bigg[& \eta^{mn} \partial_m X^I \partial_n X^I + \frac{m^2}{9} \left(X^i\right)^2 + \frac{m^2}{36} \left(X^{i'}\right)^2 - \\ & - i\psi^1 (\partial_\tau + \partial_\sigma) \psi^1 - i\psi^2 (\partial_\tau - \partial_\sigma) \psi^2 - i\frac{m}{2} \psi^2 \left(\gamma^{123} + \frac{1}{3}\gamma^4\right) \psi^1 \Bigg] . \end{aligned} \tag{3.13}$$

Let us observe that $(\gamma^{1234})^2 = 1$ and $[\gamma^{1234}, \gamma^9] = 0$, which mean that a fermionic field with definite SO(8) chirality can be decomposed according to the eigenvalues of γ^{1234}: $\psi^A = \psi^A_+ + \psi^A_-$ where $\psi^A_\pm$ are eigenstates of γ^{1234}, that is, $\gamma^{1234}\psi^A_\pm = \pm\psi^A_\pm$. With this

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decomposition, the fermion mass term in the light-cone action, eq. (3.13), can be rewritten as

$$-2i\frac{m}{3}\psi_+^2\gamma^4\psi_-^1 + 2i\frac{m}{6}\psi_-^2\gamma^4\psi_+^1 \,. \tag{3.14}$$

In our notation, fermion has the same SO(1,9) and SO(8) chirality measured by Γ^9 and γ^9, respectively. Thus, among sixteen fermionic components in total, eight with $\gamma^{12349} = 1$ have the mass of $m/6$ and the other eight with $\gamma^{12349} = -1$ the mass of $m/3$, which are identical with the masses of bosons. Therefore the theory contains two supermultiplets $(X^i, \psi_-^1, \psi_+^2)$ and $(X^{i'}, \psi_+^1, \psi_-^2)$ of (4,4) supersymmetry with the masses $m/3$ and $m/6$, respectively.

4. $\mathcal{N}$=(4,4) worldsheet supersymmetry

In this section we describe the supersymmetry in the above light-cone fixed action, eq. (3.13). The supersymmetry transformation (δ_η) for the worldsheet fields is the odd part of the supertranslation in superspace, which is read off from

$$\begin{aligned} \delta_\eta Z^M E_M^r &= 2\bar{\theta}\Gamma^r\eta \,, \\ \delta_\eta Z^M E_M^a &= \eta^a \,. \end{aligned} \tag{4.1}$$

With these relations, the supersymmetry transformation rules in the light-cone gauge, eq. (3.11), and the κ symmetry fixing condition $\Gamma^+\theta = 0$ are obtained as

$$\begin{aligned} \delta_\eta X^+ &= 0 \,, \\ \delta_\eta X^- &= \bar{\theta}^A\Gamma^-\eta^A \,, \\ \delta_\eta X^I &= \bar{\theta}^A\Gamma^I\eta^A \,, \\ \delta_\eta \theta^A &= \eta^A \,. \end{aligned} \tag{4.2}$$

For the kinematic supersymmetry $\delta_{\tilde{\eta}}$ with the parameter $\tilde{\eta}$ of eq. (2.7) satisfying $\Gamma^+\tilde{\eta} = 0$, the light-cone gauge choice $X^+ = \alpha' p^+ \tau$ is preserved and the transformation rules for the physical degrees of freedom are

$$\begin{aligned} \tilde{\delta} X^I &= 0 \,, \\ \tilde{\delta}\theta^A &= \tilde{\eta}^A \,. \end{aligned} \tag{4.3}$$

These rules tell us that the kinematic supersymmetry is nonlinearly realized on the string worldsheet.

For the dynamical supersymmetry δ_η with the parameter of Eq. (2.12), fermionic κ-symmetry fixing condition is no longer preserved, because $\Gamma^+\eta \neq 0$. Therefore we need the supplement κ transformations so that the total transformation rule,

$$\delta = \delta_\eta + \delta_\kappa$$

preserves κ-symmetry fixing condition. In the superspace notation, the κ symmetry transformation (δ_κ) satisfies the following equations

$$\begin{aligned} \delta_\kappa Z^M E_M^r &= 0 \,, \\ \delta_\kappa Z^M E_M^a &= (1 - \Gamma\Gamma^9)^a{}_b\kappa^b \,, \end{aligned} \tag{4.4}$$

where the matrix Γ is given by

$$\Gamma = \frac{1}{2\sqrt{-g}}\epsilon^{mn}\Pi^r_m\Pi^s_n\Gamma_r\Gamma_s \tag{4.5}$$

with the determinant g of the induced metric g_{mn} given by $g_{mn} = \Pi^r_m\Pi^s_n\eta_{rs}$. Γ has the properties as a projection operator:

$$\Gamma^2 = 1\,, \qquad \mathrm{Tr}\Gamma = 0\,. \tag{4.6}$$

In the component notation, the transformation rules for the κ-symmetry in our gauge choice become

$$\begin{aligned}
\delta_\kappa X^+ &= 0\,,\\
\delta_\kappa X^- &= \delta_\kappa\bar\theta^A\Gamma^-\theta^A\,,\\
\delta_\kappa X^I &= \delta_\kappa\bar\theta^A\Gamma^I\theta^A\,,\\
\delta_\kappa\theta^A &= (1-\Gamma\Gamma^9)\kappa^A\,.
\end{aligned} \tag{4.7}$$

For the superstring case, it is usually convenient to introduce

$$\kappa^A_m = -i\frac{\sqrt{-h}}{2\sqrt{-g}}\Pi_m\cdot\Gamma\kappa^A\,, \tag{4.8}$$

which allows us to view the parameter κ as a worldsheet vector. It is easily checked using Eqs. (4.5) and (4.6) that κ^2_m (κ^1_m) is the (anti-) self-dual vector and thus each worldsheet vectors has one independent component, say ρ^A. In our notation, $\kappa^{1\tau} = -\kappa^{1\sigma} = -\rho^1$ and $\kappa^{2\tau} = \kappa^{2\sigma} = -\rho^2$. Then the $\delta_\kappa\theta^A$ becomes

$$\delta_\kappa\theta^A = 2i\Pi^r_m\Gamma_r\kappa^{Am}\,. \tag{4.9}$$

Now one can find the appropriate κ transformation parameters so that the total transformation rules obey $\Gamma^+\delta\theta = 0$.[4] The resultant transformation rules are as follows:

$$\begin{aligned}
\delta X^I &= \frac{i}{\sqrt{\alpha' p^+}}\psi^A\gamma^I\epsilon^A\,,\\
\delta\psi^1 &= \frac{1}{\sqrt{\alpha' p^+}}\left(\partial_\tau X^I - \partial_\sigma X^I\right)\gamma^I\epsilon^1 + \frac{1}{\sqrt{\alpha' p^+}}\left(\frac{m}{3}X^i\gamma^{123}\gamma^i + \frac{m}{6}X^{i'}\gamma^{123}\gamma^{i'}\right)\epsilon^2\,,\\
\delta\psi^2 &= \frac{1}{\sqrt{\alpha' p^+}}\left(\partial_\tau X^I + \partial_\sigma X^I\right)\gamma^I\epsilon^2 + \frac{1}{\sqrt{\alpha' p^+}}\left(\frac{m}{3}X^i\gamma^{123}\gamma^i + \frac{m}{6}X^{i'}\gamma^{123}\gamma^{i'}\right)\epsilon^1\,,
\end{aligned} \tag{4.10}$$

where we performed the rescaling in fermions $\psi \to \psi/(2^{1/4}\sqrt{2\alpha' p^+})$ and the supersymmetry parameter $\epsilon \to \epsilon/2^{1/4}$ given in (2.12). Now it is straightforward to check that the light-cone action eq. (3.13) is on-shell invariant under the above (4,4) supersymmetry, eq. (4.10).

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[4] For example, see ref. [26].

5. Discussions

The simplicity of the action of the IIA GS superstring on the PP-wave geometry and its supersymmetry is quite striking. The light-cone action consists only of quadratic terms and thus the theory is free. Furthermore the dynamical supersymmetry is independent of the worldsheet time coordinate which implies that the fields in the same supermultiplet have the same mass. Since the number of dynamical supersymmetries is half the number of those in IIA superstring on the flat background, the full field contents split into two supermultiplets. Each supermultiplet consists of half number of bosons and fermions which has the same mass. Therefore the theory can be interpreted as the free $\mathcal{N}$=(4,4) two-dimensional theory with two massive supermultiplets.

The mode expansions are straightforward and the spectrum should have the same supersymmetry structure. It should be much more straightforward to find BPS D-branes in the string theory [27]–[31] than the original eleven-dimensional M theory. For example, all the BPS states found in [11, 12, 32] in the context of matrix model in eleven dimensions would be found in this string theory. The IIA matrix string theory on PP-wave background should also have the same supermultiplet structure as it becomes free string theory in the IR limit. In that sense, the matrix string theory on ten dimensional PP-wave seems to be more transparent than the matrix model on eleven dimensional PP-wave. All these will be presented in the forthcoming paper [33].

A. IIA PP-wave geometry

In this appendix, we describe how to get the ten-dimensional IIA PP-wave through the dimensional reduction from the eleven-dimensional PP-wave. We also give some useful formula used in our computations.

The PP-wave geometry in eleven-dimensions is given by

$$\begin{aligned} ds_{11}^2 &= -2dX^+dX^- - \left(\sum_{i=1}^{3}\frac{\mu^2}{9}(X^i)^2 + \sum_{i'=4}^{9}\frac{\mu^2}{36}\left(X^{i'}\right)^2\right)(dX^+)^2 + \sum_{I=1}^{9}\left(dX^I\right)^2 \,, \\ F_{+123} &= \mu \,, \end{aligned} \tag{A.1}$$

where μ is a parameter of the geometry and characterizes the matrix theory of ref. [8].

We change the coordinates as

$$X^+ = x^+ \,, \qquad X^- = x^- - \frac{\mu}{6}x^4x^9 \,, \qquad X^I = x^I \,, \qquad I = 1,2,3,5\ldots 8 \,, \tag{A.2}$$

$$X^4 = x^4\cos\left(\frac{\mu}{6}x^+\right) - X^9\sin\left(\frac{\mu}{6}X^+\right) , \qquad X^9 = x^4\sin\left(\frac{\mu}{6}x^+\right) + x^9\cos\left(\frac{\mu}{6}x^+\right) \tag{A.3}$$

under which the geometry becomes

$$\begin{aligned} ds_{11}^2 &= -2dx^+dx^- - \left(\sum_{i=1}^{3}\frac{\mu^2}{9}\left(x^i\right)^2 + \sum_{i'=5}^{8}\frac{\mu^2}{36}\left(x^{i'}\right)^2\right)(dx^+)^2 + \frac{2}{3}\mu x^4 dx^9 dx^+ + \sum_{I=1}^{9}\left(dx^I\right)^2, \\ F_{+123} &= \mu \,. \end{aligned} \tag{A.4}$$

Therefore in the new coordinates, $\partial_9 = \frac{\partial}{\partial x^9}$ is an isometry. If the x^9 coordinate is periodically identified, M theory in this PP-wave background can be compactified to give type-IIA string theory in the following new PP-wave geometry:

$$\begin{aligned} ds^2 &= -2dx^+dx^- - A\left(x^I\right)\left(dx^+\right)^2 + \sum_{I=1}^{8}\left(dx^I\right)^2 , \\ F_{+123} &= \mu\,, \qquad F_{+4} = -\frac{\mu}{3}\,, \end{aligned} \tag{A.5}$$

where

$$A(x^I) = \sum_{i=1}^{4}\frac{\mu^2}{9}\left(x^i\right)^2 + \sum_{i'=5}^{8}\frac{\mu^2}{36}\left(x^{i'}\right)^2 . \tag{A.6}$$

From this ten dimensional IIA geometry, we may choose the zehnbein as

$$\begin{aligned} e^+ &= dx^+ , \\ e^- &= dx^- + \frac{1}{2}A\left(x^I\right)dx^+ , \\ e^I &= dx^I . \end{aligned} \tag{A.7}$$

The only non-vanishing spin connection one-form is

$$\omega^{-I} = \frac{1}{2}\partial_I A\left(x^I\right)dx^+ . \tag{A.8}$$

The supersymmetry transformation rule for the ten-dimensional gravitino ψ_μ, in the string frame and in the vanishing fermion background, is

$$\delta_\eta \psi_\mu = \left(\nabla_\mu + \Omega_\mu\right)\eta\,, \tag{A.9}$$

where the covariant derivative is given by

$$\nabla_\mu = \partial_\mu + \frac{1}{4}\omega_\mu{}^{rs}\Gamma_{rs}$$

and

$$\begin{aligned} \Omega_\mu =& -\frac{1}{8}\Gamma_r\Gamma_s\eta e_\mu{}^r e^{\nu s}\partial_\nu\phi - \frac{1}{64}e_\mu{}^r(\Gamma_{rst} - 14\eta_{rs}\Gamma_t)\Gamma^{11}\eta e^\phi F^{st} + \\ &+\frac{1}{96}e_\mu{}^r\left(\Gamma_r{}^{stu} - 9\delta_r^s\Gamma^{tu}\right)\Gamma^{11}\eta H_{stu} + \frac{1}{768}e_\mu{}^r\left(3\Gamma_r{}^{stuv} - 20\delta_r^s\Gamma^{tuv}\right)\eta e^\phi F'_{stuv} \end{aligned} \tag{A.10}$$

The dilatino field transforms under the supersymmetry as

$$\begin{aligned} \delta_\eta\lambda =& -\frac{1}{2\sqrt{2}}\Gamma^r\Gamma^{11}\eta e_r{}^\nu\partial_\nu\phi - \frac{3}{16\sqrt{2}}\Gamma^{rs}\eta e^\phi F_{rs} + \\ &+\frac{1}{24\sqrt{2}}\Gamma^{rst}\eta H_{rst} + \frac{1}{192\sqrt{2}}\Gamma^{rstu}\Gamma^{11}\eta e^\phi F'_{rstu} . \end{aligned} \tag{A.11}$$

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References

[1] J. Kowalski-Glikman, *Vacuum states in supersymmetric Kaluza-Klein theory*, *Phys. Lett.* **B 134** (1984) 194.

[2] R. Gueven, *Plane wave limits and t-duality*, *Phys. Lett.* **B 482** (2000) 255 [hep-th/0005061].

[3] J. Figueroa-O'Farrill and G. Papadopoulos, *Homogeneous fluxes, branes and a maximally supersymmetric solution of M-theory*, *J. High Energy Phys.* **08** (2001) 036 [hep-th/0105308].

[4] R. Penrose, *Any space-time has a plane wave limit*, in *Differential geometry and gravity*, Reidel, Dordrecht 1976, pp. 271.

[5] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, *Class. and Quant. Grav.* **19** (2002) L87 [hep-th/0201081];
M. Blau, J. Figueroa-O'Farrill and G. Papadopoulos, *Penrose limits, supergravity and brane dynamics*, *Class. and Quant. Grav.* **19** (2002) 4753 [hep-th/0202111].

[6] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A new maximally supersymmetric background of IIB superstring theory*, *J. High Energy Phys.* **01** (2002) 047 [hep-th/0110242].

[7] R.R. Metsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044];
R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave Ramond-Ramond background*, *Phys. Rev.* **D 65** (2002) 126004 [hep-th/0202109].

[8] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[9] K. Dasgupta, M.M. Sheikh-Jabbari and M. Van Raamsdonk, *Matrix perturbation theory for M-theory on a PP-wave*, *J. High Energy Phys.* **05** (2002) 056 [hep-th/0205185].

[10] G. Bonelli, *Matrix strings in PP-wave backgrounds from deformed super Yang-Mills theory*, *J. High Energy Phys.* **08** (2002) 022 [hep-th/0205213].

[11] S.-J. Hyun and H.-J. Shin, *Branes from matrix theory in PP-wave background*, *Phys. Lett.* **B 543** (2002) 115 [hep-th/0206090].

[12] D.-S. Bak, *Supersymmetric branes in PP wave background*, hep-th/0204033.

[13] N.-W. Kim and J. Plefka, *On the spectrum of PP-wave matrix theory*, *Nucl. Phys.* **B 643** (2002) 31 [hep-th/0207034].

[14] K. Dasgupta, M.M. Sheikh-Jabbari and M. Van Raamsdonk, *Protected multiplets of M-theory on a plane wave*, *J. High Energy Phys.* **09** (2002) 021 [hep-th/0207050].

[15] N. Kim and J.-H. Park, *Superalgebra for M-theory on a PP-wave*, hep-th/0207061.

[16] N.-W. Kim, K.-Y. M. Lee and P. Yi, *Deformed matrix theories with $N = 8$ and fivebranes in the PP wave background*, hep-th/0207264.

[17] K. Sugiyama and K. Yoshida, *Supermembrane on the PP-wave background*, *Nucl. Phys.* **B 644** (2002) 113 [hep-th/0206070].

[18] J. Michelson, *(twisted) toroidal compactification of PP-waves*, *Phys. Rev.* **D 66** (2002) 066002 [hep-th/0203140].

[19] M. Alishahiha, M.A. Ganjali, A. Ghodsi and S. Parvizi, *On type-IIA string theory on the PP-wave background*, hep-th/0207037.

[20] I. Bena and R. Roiban, *Supergravity PP-wave solutions with 28 and 24 supercharges*, hep-th/0206195.

[21] J. Michelson, *A PP-wave with 26 supercharges*, hep-th/0206204.

[22] K. Sugiyama and K. Yoshida, *Type IIA string and matrix string on PP-wave*, *Nucl. Phys.* **B 644** (2002) 128 [hep-th/0208029].

[23] M.J. Duff, P.S. Howe, T. Inami and K.S. Stelle, *Superstrings in $d = 10$ from supermembranes in $D = 11$*, *Phys. Lett.* **B 191** (1987) 70.

[24] B. de Wit, K. Peeters, J. Plefka and A. Sevrin, *The M-theory two-brane in $AdS_4 \times S^7$ and $AdS_7 \times S^4$*, *Phys. Lett.* **B 443** (1998) 153 [hep-th/9808052];
P. Claus, *Super M-brane actions in $AdS_4 \times S^7$ and $AdS_7 \times S^4$*, *Phys. Rev.* **D 59** (1999) 066003 [hep-th/9809045].

[25] M. Cvetič, H. Lu, C.N. Pope and K.S. Stelle, *T-duality in the Green-Schwarz formalism and the massless/massive IIA duality map*, *Nucl. Phys.* **B 573** (2000) 149 [hep-th/9907202].

[26] S. Hyun and H. Shin, *Supersymmetry of Green-Schwarz superstring and matrix string theory*, *Phys. Rev.* **D 64** (2001) 046008 [hep-th/0012247].

[27] A. Dabholkar and S. Parvizi, *Dp-branes in PP-wave background*, *Nucl. Phys.* **B 641** (2002) 223 [hep-th/0203231].

[28] K. Skenderis and M. Taylor, *Branes in AdS and PP-wave spacetimes*, *J. High Energy Phys.* **06** (2002) 025 [hep-th/0204054].

[29] A. Kumar, R.R. Nayak and Sanjay, *D-brane solutions in PP-wave background*, *Phys. Lett.* **B 541** (2002) 183 [hep-th/0204025].

[30] M. Alishahiha and A. Kumar, *D-brane solutions from new isometries of PP-waves*, *Phys. Lett.* **B 542** (2002) 130 [hep-th/0205134].

[31] A. Biswas, A. Kumar and K.L. Panigrahi, *p-p' branes in PP-wave background*, hep-th/0208042.

[32] M. Alishahiha and M. Ghasemkhani, *Orbiting membranes in M-theory on $AdS_7 \times S^4$ background*, *J. High Energy Phys.* **08** (2002) 046 [hep-th/0206237].

[33] S. Hyun and H. Shin, in preparation.